A Sparse Grid Hexagonal Grid on the sphere based on the L-Galerkin approach o2o2

Jurgen Steppeler*†[1]

[1]Universität Bonn = University of Bonn – Germany

Abstract

Hexagonal and triangular meshes belong to the same family and are for example used in the models MPAS and ICON. While ICON and MPAS use one grid point per cell, we here consider developments on such grids using L-Galerkin methods. Such L-Galerkin methods use more than one grid point per cell. So far such L-Galerkin methods exist only on quadrilateral cells for realistic models. The spectral element model SE3 use the full grid on rhomboidal icosahedral grids. Stating from rhomboidal cells on the sphere, a triangular cell model is equivalent, when each rhomboidal cell is divided into two triangles. The mentioned L Galerkin methods SE3 from Boulder and Monterey use the full grid. Hexagonal grids can be obtained from triangular meshes by combining 6 triangles to a hexagon. With full grids, hexagons are just rearranged quadrilaterals or triangles. Basis function methods, however, adjust the basis functions to the cells. This means that triangular, rhomboidal and hexagonal cells have different sets of basis functions. Some of them may be of 9th order, if third order full grid basis functions are used. They make a contribution below roundoff error to the computation of a field. When analysing classical spectral models, for example, often many amplitudes are actually observed to be below roundoff error. They have a potential to create difficulties, as in high resolution models a large number of them are added to create significant errors. The idea of sparse grids is to avoid such basis functions of 9th order within a 3rd order method. For classic Galerkin methods, sparse grids are a possibility. The same is true for L-Galerkin methods. While SE3 uses the full grid, the L-Galerkin method o3o3, for example, uses the sparse grid and such models have been tested in two dimensions on a plane with hexagons and rhomboids. Hexagonal grids have a sparse grid with (in 3-d) only 1/10th of the points of the full grid with a corresponding potential to save CPU time by a factor of 20. Such sparse hexagonal grids have been realised in a plane. The author together with J. Li currently works on building a sparse hexagonal grid on the sphere with a realistic model using MPAS physics. It has been pointed out that when the grid is irregular and adapts to mountains in three dimensions, this creates a rather good approximation to mountain generated flows, called the cut cell approximation. Practical realisations of such methods have run into difficulties, which some researchers led to the opinion that cut cell would not work with meteorological models. It is certainly necessary for cells at the boundary of the model area (at mountains) to pose boundary conditions. Experiments will be shown to indicate that two sets of boundary values are necessary: one is for the fast wave part of the equations and the other for the advection. The boundary values described in the literature are OK for the fast waves, while the advection needs the open boundary values at outflow points. It may be mentioned that errors generated by not using open boundary conditions for

*Speaker

†Corresponding author: jsteppeler@t-online.de

the advection can be spectacular. Many of the stated advantages occur already with second order basis functions. It is shown by examples, that some established models converge in less than 1st order. The mentioned development therefore starts with second order basis functions for hexagons as a first step.

Keywords: L, Galerkin, sparse grids