
A scalable ADER-DG transport method with a polynomial-order-independent CFL limit for efficient high-order simulations

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Abstract

Transport methods play a central role in geophysical fluid models, driving the need for approaches that are efficient, scalable, conservative, and high-order accurate. Discontinuous Galerkin (DG) and ADER-DG methods are popular due to their scalability, conservation properties, and high-order accuracy. However, as the polynomial order increases, these methods are constrained by strict time step limits, leading to high computational costs. In this work, we introduce a novel, scalable ADER-DG transport method that overcomes this limitation and features an element-size CFL condition, allowing the maximum stable time step to be independent of the polynomial order. Our method employs a series of element-local implicit problems to optimize the domain of dependence, achieving stable CFL values (with respect to the element size) of 1 in 1D, 0.67 in 2D, and 0.58 in 3D. For example, at 5th order, this results in a ~ 20 -fold increase in time step compared to standard DG methods. We rigorously prove the maximum CFL limit for 1D and derive the 2D and 3D values through a semi-analytical von Neumann stability analysis. The method's conservation and convergence are validated through standard test cases on the plane and the sphere. The method's conservation and convergence are validated through standard test cases on both the plane and sphere. Additionally, we demonstrate the method's robustness in simulating geostrophic turbulence.

Keywords: Transport, High order, Scalable, discontinuous Galerkin, ADER, DG

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