Thermodynamic consistency and energy conservation for the multiphase moist compressible Euler equations

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Abstract

The concepts of thermodynamic consistency via the use of a potential function for thermodynamic processes, and energy conservation via the preservation of Hamiltonian structure, are harmonised for a multiphase (dry, vapour, liquid, ice) model of the compressible Euler equations.

The Hamiltonian structure of the discrete system is determined by casting the prognostic equations for velocity, density, entropy and total moisture in skew-symmetric form, with the tendencies determined via the variational derivatives of the thermodynamic potential (specific internal energy) with respect to the prognostic variables. The fractions of the vapour, liquid and ice phases are then determined instantaneously under the assumption of thermodynamic equilibrium (no entropy generation) by the solution of a local Newton problem subject to the equality of the Gibbs free energies of the different phases.

This formulation is implemented within the context of a discontinuous-Galerkin method, with the flux terms and their energy conserving pressure gradient adjoints formulated so as to conserve or provably damp tracer variances as desired for the entropy and total moisture. This energy conserving, tracer variance bounding flux formulation is sufficient to stabilise the model for the flow regimes of a three phase moist bubble and forced convective turbulence for long simulation times.

Keywords: Thermodynamic consistency, structure preservation, moist compressible Euler equations

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